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Integrating Mathews Stability Chart into the Stope Layout Determination Algorithm

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Abstract

Stope layout optimization is an important feature of underground mining that maximizes the economic value of the project while taking mining limits into account. The large number of parameters and constraints makes it difficult to obtain the optimum condition. Several algorithms have been created to address these problems using a variety of methods. However, the circulating method has not explicitly included stope dimension stability analysis, resulting in a solution that is not stability-proven, which can result in a suboptimal solution. This study integrates the Mathews stability graph into the stope optimization algorithm so that the optimized stope layout considers stability conditions directly through an assessment of the available geomechanical data within the block model. The proposed algorithm is validated through a case study of a synthetic block model created by considering variations in grade and the geomechanical conditions of the rock. Furthermore, several scenarios are created to compare the performance of the algorithm that applies variations in stope sizes with the common case study of stope sizes that remain fixed. A more detailed assessment is also conducted on each final stope layout wall to ensure the successful application of stability analysis in the proposed algorithm through back analysis on the Mathews stability graph. The optimization results show that all walls in the final stope layout fall into the stable condition. Also, the proposed algorithm is also capable of maintaining the project's economic value. Ultimately, the proposed algorithm can be deemed applicable and suitable for use in the initial stages of mining as a comprehensive assessment of the optimal stope layout, taking into account the stability conditions of the stope.

Keywords Stope optimization · Heuristic algorithm · Mathews Stability Chart · Stope layout · Underground mine

1 Introduction

Underground stope stability analysis methods have been widely developed using various techniques such as empirical [1], analytical [2, 3], and numerical [4, 5] in line with the increasing complexity of rock conditions with deeper mining. The increasing complexity of underground mines

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creates challenges regarding stope design. Poorer rock conditions result in smaller stopes, which then limit reserve, while better rock conditions tend to accommodate bigger stope dimensions [6]. To accommodate the complexity and further simplify the stability analysis, Critical Span Graphs [7] and Mathews Stability Chart [8–10] are two of the empirical approaches that are still widely used as industry standards. These approaches could help engineers determine the stability of stope designs faster, thus making the generation of specific stope designs in certain geomechanical conditions possible. While varying stope dimensions to their specific geomechanical properties could have significant impacts on mine reserves, thus creating more value in mine projects, stope design is often limited in a conservative way, such as when poorer geomechanical data is chosen as the basis for the stope dimension. Furthermore, research integrating stability topics with stope optimization is still limited while such studies are imperative [11].

Many stope optimization algorithms have been developed to assist engineers in solving optimization problems [12–14]. Various optimization techniques, such as stochastic [15–17], exact algorithms, and heuristic algorithms [18–20], have been used to ensure that the stope optimization results in maximum NPV with stope dimensions as constraints. The exact algorithm is formed from a mathematical model, ensuring the best solution is obtained. Some algorithms falling into this category include Branch and Bound [21], Dynamic Programming [22], and Downstream Geostatistical Approaches [23]. Dynamic Programming stope optimization was introduced by Riddle [22] in a block-caving case study, which also has its weaknesses due to limitations in its application to that method. Deraime et al. [23] introduced the downstream geostatistical approach to determine parts of the ore body with the best economics. However, the application of this algorithm is limited to cut-and-fill or sublevel stoping methods. The solutions generated by this algorithm cannot yet be considered optimal as they have not been proven with practical mining designs. Ovanic and Young [21] further developed the Branch and Bound algorithm applied to integer programming. Generally, Integer programming requires considerable resources for problem-solving, making it often unfeasible for large case studies. The integration with the Branch and Bound algorithm allows problems to be broken down into smaller ones, making the problem-solving process more efficient. Nevertheless, the effectiveness of solving problems in large case studies remains a weakness, so this algorithm has not been applied beyond one-dimensional case studies.

Contrary to exact algorithms, heuristic algorithms do not focus on mathematical models; thus, the solutions they generate do not fully achieve the global optimum but are close enough to the global optimum. Some algorithms falling into this category include Octree Division [24], Floating Stope [25], Multiple Pass Floating Stope [26], Maximum Neighborhood [27], Topal and Sens [28], and Sandanayake [29]. The Octree Division algorithm introduced by Cheimanoff et al. [24] is capable of working in three-dimensional case studies where the "optimum" part of the block model is determined based on mining constraints and economics. In practice, this algorithm approaches the optimal condition by producing a 3D stope layout. However, the structure of the algorithm, which allows waste blocks to enter the final stope layout, reduces the economic value of the final stope layout. Hence, the optimal solution has not been achieved yet. The next development in stope algorithms, which is quite applicable and adopted in commercial software, is the Floating Stope by Alford [25]. The approach used is similar to other algorithms in open-pit case studies, such as the Floating Cone. Similar to the Floating Cone, one advantage of this algorithm is its simplicity, where a stope of predetermined dimensions is floated on the block model, and an

assessment of the stope is conducted to determine its economic feasibility. However, a weakness of this algorithm lies in not considering the important concept of overlapping stopes. Overlapping stope solutions result in double-counted reserves, leading to increased economic feasibility. Adjustments have to be made to ensure that the mined material truly represents the actual mine and its economic value. The need for manual intervention in this algorithm means that the solution from the Floating Stope algorithm cannot yet be considered optimal. To address this weakness, Cawrse [26] developed the Multiple Pass Floating Stope, providing additional information to engineers and making the assessment of the Floating Stope output easier. However, the main weakness of the overlapping stope concept has not been resolved, causing this algorithm to still be unable to produce an optimal solution. Still based on the principle of the Floating Stope, the Maximum Neighborhood algorithm was developed by Ataee-Pour [27]. A more detailed approach, aggregating blocks into stope shapes and, in the process, eliminating blocks with negative economic value, makes this approach better. However, the solutions generated are highly dependent on the initial location of the optimization iterations, causing this algorithm to not yet produce an optimal solution. Later, the heuristic approach developed by Topal and Sens [28] changes geological blocks into economic blocks of uniform size and then forms stopes of specific dimensions in each block model, assessing the stope attributes to see their feasibility. One breakthrough of this approach is the final output of the stope in three dimensions. The structure of the algorithm that sequentially eliminates sets of stopes is a weakness of this approach, making the optimal stope layout not necessarily achievable. Bai [30] developed a heuristic algorithm applied to the sublevel mining method. The limitation of this approach lies in the mining method's conditions and its application, which can only be applied to small ore bodies. Finally, Sandanayake [29] developed a heuristic algorithm by modifying the Floating Stope, where first, the block economic value (BEV) is determined by calculating all economic and geological components within the block. Then, stopes of specific sizes are floated within the block model, while the economic value of the stope is calculated based on the cumulative BEV values entering the stope. Elimination is then carried out on stopes with negative economic value, while sets are formed on stopes with positive values that do not overlap. The set of stopes with the best economic value is chosen as the best solution. However, calculating BEV at the beginning of optimization is one of the weaknesses of this algorithm because economic parameters are independent of mining scenarios, thus the possibility of hidden positive economic value stopes not being further assessed.

The early stope optimization algorithm presented has a common way to express stable stope dimension. The stope,

as a mineable area, is typically simplified into a box-shaped dimension with floating width, length, and height [26, 31, 32]. To address practical requirements, dimensional constraints were implemented to ensure that the optimization outcomes met geotechnical and technical conditions [33, 34]. Dimension considerations in optimization are focused on two approaches: fixed dimensions [29, 31] and variable dimensions [35–38]. Fixed dimensions impose uniform, predefined stope dimensions at the initial optimization stage. This constraint limits the algorithm's flexibility in selecting the best stope due to the predetermined size set by the user at the start of optimization. Meanwhile, variable dimensions were applied by setting the maximum and minimum dimension constraints allowed for the stope layout. By providing maximum and minimum dimension constraints, the optimized stope layout fulfills both geomechanical and operational considerations. However, both approaches require users to determine the generally allowable stope size in each optimization domain. Furthermore, variations in rock conditions are not directly considered in the optimization algorithm.

The use of stability analysis in stope optimization algorithm is still limited, used separately from optimization steps, where the most pessimistic geomechanical data is usually used as the basis for determining mining design in a wider area. Limited geomechanical data provides a large amount of uncertainty and eliminates economic potential, as some areas may have marginal value when mined with different stope dimensions. The latest study that adopted stability analysis in optimization algorithms was conducted by Esmaeili et al. [39] by applying stability analysis to the Caving Graph [40] as mining constraints combined with a network flow algorithm. The algorithm was successfully applied to the sublevel caving mining method with limited block numbers. As stope stability analysis methods are widely available, the potential of integrating stability analysis with currently available optimization algorithms is significant. The advantage of this methodology lies in the ability of the algorithm to read and analyze geomechanical data that is already quantitatively available to create stope dimension recommendations. This study aims to integrate Mathews stability analysis [41] into a mining optimization algorithm [32].

2 Proposed Algorithm

Stope designs that represent variations of rock conditions are needed to maximize the project values. In this study, the stope optimization algorithm [31] was modified by adding a stage of stope dimension recommendation based on the Mathews stability graph [41], making the overall algorithm stages as shown in Fig. 1. The approach was carried out by calculating the stability number (*N*) based on the geomechanical parameters available in the block model, which include *factor A*, *factor B*, *factor C*, and *Q' value*. *N numbers* are generated throughout iteration based on the available stopes walls that are constrained within the ore body. Further, the maximum allowable hydraulic radius is determined to limit the maximum stope dimension in each block location in the block model. Optimization was then carried out on a similar basis, but with the addition of the geomechanical constraint that was newly proposed.

In this study, the application of the proposed algorithm was limited to the open-stope or sublevel method for metal mines, as the Mathews Stability Chart suggested. Further, consideration of dip angle, thickness, fault, and aquifer of the ore body was considered in the parameters utilized in the Mathews Stability Chart that were already presented in the block model in the form of *factor A*, *B*, *C*, and *Q'* value, while stope wall orientation was limited to vertical as the base of the algorithm was limited to [31].

As for the mathematical models become very complex, Table 1 summarizes the notations, parameters, and decision variables that were used for the subsequent sections.

2.1 Objective Function

The objective function of this study is to maximize the stope economic value by accumulating the block economic value inside the optimum stopes. This was done by utilizing Eq. (1). In order to determine the stopes economic value, two main parameters were used: the geological parameter, including metal grade, and the economic parameter, including metal price and cost components. The block value is calculated in some sequences. First, block tonnage was determined by block lengths and rock density via Eq. (2). After the tonnage values of the blocks are known, the economic value of the block is calculated using Eq. (3) by applying economic parameters such as commodity price, mining cost, processing cost, and selling cost.

$$MAX \sum v_r \times tags \tag{1}$$

$$tr = Ho \times Lo \times Wo \times do \tag{2}$$

$$v_r = \left[\left(p - r_r \right) \times g_r \times y - \left(e_r + c_r \right) \right] \times t_r \tag{3}$$

2.2 Stope Height Constraint

The stope height constraint limits the maximum optimized stope height by ensuring that the cumulative height of mining blocks on the *z*-axis ($nzs_{i,j,k}$) does not exceed the allowable stope height. Stope height is determined by considering

Fig. 1 General algorithm steps



the mining method that is applied in the area and set by the user. Allowing one axis to be fixed decreases the complexity of the algorithm as it will only optimize the stope length and span. However, full consideration needs to be given by the user, as the stope height will also dictate the mining level. A shorter stope will generate many levels and further impact the need for mine access while also creating the opportunity to do selective mining. On the contrary, a higher stope will generate fewer levels, but the production rate could be higher, further impacting the production cost. This condition is displayed in Fig. 2 where the stope's origin, positioned at the lowest elevation of the block model (marked by the green-colored box), is the determining block for the stope's height constraint $(nzs_{i,i,k})$, which also defines the number and location of levels. Nevertheless, the final layout will be driven by the rock conditions, as this constraint only enforces one of the three axes.

In the proposed algorithm, the stope shape is controlled via block quantity relative to its axis. Thus, conversion from allowable stope height to allowable mining block is needed. Equation (4) implies that the maximum stope height is converted by dividing the maximum height by the block height. The calculation was possible because of the regularity of the block size.

$$nzs_{i,j,k} = Hmax/Ho \tag{4}$$

2.3 Maximum and Minimum Width Constraint

The maximum and minimum constraints limit the stope size during the optimization process, ensuring that the optimal stope meets operational and geomechanical criteria. The dimensions of mining equipment are considered the minimum operational width defined by the user (*Wmins*). The stope's width should accommodate the equipment size operating in that area. The use of mechanical equipment tends to require a larger minimum width for the stope compared to traditional mining. In some cases within narrow veins, the equipment width may conflict with the vein width, necessitating a wider stope to compensate for the mechanized mining activities in that location, further impacting the increase in planned dilution, thus reducing the economic feasibility of the stope.

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 Table 1
 List of notations for the mathematical models and methodology

Differing from the minimum width (Wmins), the minimum stope length (Lmins) is typically determined based on the length of the stope advancement, where its width is no

smaller than the minimum stope advancement length. This constraint ensures that no stope design is created smaller than the stope advance length.

A significant factor affecting the maximum stope width and length (*Wmaxs* and *Lmaxs*) is geomechanics. Solid rock, limited water presence, and favorable stress conditions are indicators of favorable rock conditions where stope sizes can generally be larger to meet production needs. Furthermore, in the design aspect, the orientation of the structure and stope walls can be a determining factor for stability/safety in stope design. Mathews [42] proposed an empirical approach applicable to open stopes or sublevel stoping, where the hydraulic radius and stability number (N') are used as indicators for the maximum stable stope dimensions. The application and integration of the geomechanical constraint model into the stope dimension constraints are explained in more detail in Section 4.

Geomechanical constraints are established by ensuring that the stopes have dimensions smaller than the allowed hydraulic radius at the location where the stopes will be formed. Meanwhile, operational constraints ensure that the dimensions of the stopes are larger than the minimum allowed dimensions at a block model location. Both of these constraints are combined in a unified constraint that regulates the maximum and minimum dimensions along the *x*-axis ($nxs_{i,j,k}$), *y*-axis ($nys_{i,j,k}$), and *z*-axis ($nzs_{i,j,k}$). The stope size is limited by Eq. (5) to (6), which add up the indices of the mined blocks (tags) in the stope layout and compare them to the stope size limits. Equations (5), (6), and (7), respectively, operate on the *x*-axis, *y*-axis, and *z*-axis.

$$nxs_{i,j,k} \ge \sum_{i=1}^{I} \sum_{j=1}^{j} \sum_{k=1}^{k} tags \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(5)

$$nys_{i,j,k} \ge \sum_{i=1}^{i} \sum_{j=1}^{J} \sum_{k=1}^{k} tags \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(6)

$$nzs_{i,j,k} \ge \sum_{i=1}^{i} \sum_{j=1}^{j} \sum_{k=1}^{K} tags \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(7)

The application of these equations serves as a constraint in the stope optimization phase, as depicted in Fig. 3. The green-colored blocks depict the block origin's position where the stope dimension constraints are applied, while the dashed red lines represent the boundary of the stope layout's location with the application of stope dimension constraints. With the integrated application of geomechanical considerations in the stope dimension constraint at each stope location, the stope dimensions can be deemed representative as they meet the rock conditions.



Fig. 2 Stope height constraint

Fig. 3 Stope dimension constraint

2.4 Stope Overlapping Constraint



"Overlapping stopes" is a condition where the optimized stope layouts intersect with each other [33]. This condition

arises due to the formation of another optimal stope shape

in a nearby location. Overlapping stope results in repeated

calculations of volume and tonnage, which then raises the

value of the mined material and leads to an overly optimistic assessment of the project's feasibility. Figure 4 depicts

an illustration of overlapping stopes where the red-colored

Fig. 4 Overlapping stopes

blocks represent the area where both stopes intersect. In this case, the material attributes within the red blocks will be counted twice, potentially resulting in inaccurate mined material and economic calculations for both stopes.

In this study, the overlap constraint is applied by utilizing the mined block index (*tags*) assigned to each block falling within a stope. The stope layout is deemed feasible when, during stope determination, all blocks within that stope have a mined block index (*tags*) equal to 0. This constraint application ensures that no stope can form in that location if even a single block has a mined block index (*tags*) equal to one. Equation (8) shows the mathematical form of the stope overlap constraint, where $tags_{i,i,k}$ is the mined block indexes.

$$fu1_{i,j,k} = \begin{cases} 0 iftags_{i=1,j=1,k=1}^{i+nxs_{i,j,k},j+nys_{i,j,k},k+nzs_{i,j,k}} = 1\\ 1 otherwise \end{cases} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\} \end{cases}$$
(8)

3 Maximum Width and Span by Mathews Stability Chart

3.1 Mathews Stability Chart

Mathews [42] introduced a stability graph based on 26 cases collected from open-stope underground mining. This data was later supplemented and recalibrated by Potvin [43], which became widely used in the industry as the basis for mine planning that considers rock geomechanics conditions. The stability graph represents a plot of the stability number (N') against the shape factor (S) or also known as the hydraulic radius (HR). The calculation of N' is done by considering Rock Quality Designation (*RQD*), joint set number (J_n) , joint roughness number (J_r) , joint alteration number (J_a) , stress factor (A), joint orientation factor (B), and gravity factor (C) through Eq. (9). Meanwhile, HR is generally the ratio between the area and the perimeter, which is determined based on the length (L) and width (W) of the stope wall, as shown in Eq. (10). Both of these variable results are plotted on the stability graph to determine the stability condition of the wall through three zones depicted on the graph: stable, unstable, and cave, as seen in Fig. 5.

$$N\prime = \left(\frac{RQD}{J_n}\right) \times \left(\frac{J_r}{J_a}\right) \times A \times B \times C \tag{9}$$

$$HR = \frac{W \times L}{2 \times (W \times L)} \tag{10}$$

This study employs the stability graph developed by Potvin [43] and Nickson [44] as a stability analysis tool within



Fig. 5 Potvin-modified Mathews stability graphs [43]

the optimization algorithm. Equation (11) represents the boundary between the stable and unstable areas on the stability graph which was statistically calculated by Nickson [44] based on 175 case studies of stope stability in Potvin [43]. The stope wall dimensions allowed fall within the area above this boundary line. Through the use of this stability graph, it is also assumed that the stopes used in this algorithm are unsupported.

$$HR = 10^{(0.573 + 0.338 \log N)} \tag{11}$$

3.2 Mathews Stability Chart Application in Stope Dimensional Constraint

In this study, improvements to the existing stope optimization algorithm were made by incorporating stability analysis using Mathews Stability Chart into the algorithm as dimensional constraints. Thus, the proposed algorithm has the ability to directly address rock conditions. This was done by assessing the Mathews attribute data provided in the block model. The analysis is conducted at block locations by iterating steps as follows:

- 1. Assessing the maximum of each stope wall domain by looking for the ore domain.
- 2. Calculate the *N* stability number based on *Q*' value, factor *A*, factor *B*, and factor *C* within the wall domain.
- 3. Determine the stable condition for the wall domain by correlating the hydraulic radius and the *N stability number*.
- 4. Assessing a smaller domain until a stable condition is met
- 5. Determine the allowable stope wall dimensions by choosing the lowest hydraulic conditions between each wall.

The algorithm steps are handled by several equations, as follows: Eqs. (12) and (13) are used for the first step, to determine which part of the rock is the ore body, so that subsequent iterations of the equation will be limited to the controlled by the *i*, *j*, and *k* indices of the block. The difference between the two equations lies in the orientation in which each equation is applied. Equation (12) is utilized on the stope wall oriented towards the east, while Eq. (13) is applied on the stope wall oriented towards the north. The calculation of the *N* stability number in the second step is performed by utilizing Eqs. (14) to (17). Each of the equations represents the different calculations performed for the four walls. As seen in the equations, all four equations have different block location indices, representing calculations for block data in different domains of the stope walls. The N stability number for each of the stope wall domains is then used to calculate the allowable hydraulic

(13)

(15)

radius for the corresponding stope wall via Eqs. (18) to (21). Each of the *hydraulic radius* equations corresponds to the wall where it belongs, as the indices specifically emphasize where the calculation is performed.

$$nxs_{i,j,k} = \sum_{i=1}^{Wmax/Wo} \sum_{j=1}^{j} \sum_{k=1}^{k} tagr \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$

 $nys_{i,j,k} = \sum_{i=1}^{i} \sum_{j=1}^{Lmax/Lo} \sum_{k=1}^{k} tagr \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$

 $N1r_{i,j,k} = MINNr_{i=1,j=1,k=1}^{i,j+\frac{Lmax}{L_o},nzs} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$

$$N2r_{i,j,k} = MINNr_{i=i+(\frac{Wmax}{Wo}),j=1,k=1}^{i+\frac{Wmax}{Wo},nzs} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$

$$N3r_{i,j,k} = MINNr_{i=1,j=1,k=1}^{i+\frac{Waux}{Wo}, j, n, zs} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(16)

$$N4r_{i,j,k} = MINNr_{i=1,j=1,k=1}^{i+\frac{Wmax}{W_0}, j+\frac{Imax}{L_0}, nzs} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(17)

(12)

$$HR1r_{i,j,k} = 10^{(0.573+0.388 \times \log(N1r_{i,j,k}))} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(18)

$$HR2r_{i,j,k} = 10^{(0.573+0.388 \times \log(N2r_{i,j,k}))} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(19)

$$HR3r_{i,j,k} = 10^{(0.573+0.388 \times \log(N3r_{i,j,k}))} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(20)

$$HR4r_{i,j,k} = 10^{(0.573+0.388 \times \log(N4r_{i,j,k}))} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(21)

The selection of the lowest hydraulic radius is then performed on opposing walls to ensure that the lowest value to be used is indicated by Eqs. (22) and (23). The allowable length for stope walls is then determined based on the *hydraulic* *radius* of the corresponding wall through Eqs. (24) and (25). In the last stage, Eqs. (26) and (27) are making sure that the allowable length for the stope wall has already met operational constraints.

$$HR1r_{i,j,k} = \begin{cases} HR1r_{i,j,k} if HR1r_{i,j,k} < HR2r_{i,j,k} \\ HR2r_{i,j,k} otherwise \end{cases} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\} \end{cases}$$
(22)

$$HR3r_{i,j,k} = \begin{cases} HR1r_{i,j,k} if HR3r_{i,j,k} < HR4r_{i,j,k} \\ HR4r_{i,j,k} otherwise \end{cases} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\} \end{cases}$$
(23)

$$nys1_{i,j,k} = \frac{2HR1r_{i,j,k} \times nzs_{i,j,k} \times Ho_{i,j,k}}{(nzs_{i,j,k} \times Ho_{i,j,k}) - 2HR1r_{i,j,k}} \times Lo_{i,j,k} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(24)

$$nxs1_{ij,k} = \frac{2HR3r_{ij,k} \times nzs_{ij,k} \times Ho_{ij,k}}{(nzs_{i,j,k} \times Ho_{i,j,k}) - 2HR3r_{i,j,k}} \times Lo_{i,j,k} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\}$$
(25)

$$nxs_{i,j,k} \begin{cases} 0 if nxs_{i,j,k} > \frac{Lmins}{Lo} \\ nxs1_{i,j,k} if nxs_{i,j,k} > nxs1_{i,j,k} > \frac{Lmins}{Lo} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\} \\ nxs_{i,j,k} otherwise \end{cases}$$
(26)

$$nys_{i,j,k} \begin{cases} 0 if nys_{i,j,k} > \frac{Wmins}{Lo} \\ nys1_{i,j,k} if nys_{i,j,k} > nys1_{i,j,k} > \frac{Wmins}{Lo} \forall i \in \{1 \dots I\}, j \in \{1 \dots J\}, k \in \{1 \dots K\} \\ nys_{i,j,k} otherwise \end{cases}$$
(27)

4 Optimization

Nhelko A [45] emphasizes the limitations of exact algorithm application in large-scale cases such as stope optimization. Although the resulting solutions may achieve the global optimum, the problem-solving time typically increases exponentially with problem complexity, making this algorithm category infeasible for large-scale cases. Heuristic algorithms are commonly employed solutions for addressing complex problems like stope optimization, ensuring fast problem-solving while still focusing on optimization objectives. This study applies heuristic algorithms in optimization techniques, enabling large-scale cases to serve as a benchmark for algorithm validation. One of the previously developed heuristic algorithms is Sandanayake [29, 31]. Sandanayake [29] introduces a heuristic algorithm for stope optimization with the following general steps:

- 1. Initialization of all data, parameters, and variables
- 2. Stope formation through mining block aggregation
- 3. Update of stope attributes based on mined blocks
- 4. Extraction of subsets with economic values greater than 0
- 5. Identification of overlapping stopes
- 6. Creation of a set containing non-overlapping stopes
- 7. Calculation of economic value for each non-overlapping stope set
- 8. Selection of sets and determination of stopes with the highest economic value.



Fig. 6 Block model for Au grade

The algorithm was modified by incorporating Mathews' stability considerations into the stope dimension constraints (*nys*, *nxs*, *nzs*). The varying dimension constraints are held by each block within the block model in line with the geomechanical conditions of that block. Hence, the stope dimensions will vary according to the geomechanical conditions at that location. These constraint applications are implemented in the initial stage of stope creation. The subsequent stage remains relevant, where non-overlapping stopes are created, and ultimately, the stope set with the highest economic value is selected.

5 Case Study

To test the validity of the algorithm proposed in this study, a block model was created by considering several case studies of underground gold mines in Indonesia. Prasetyo et al. [46] modeled a gold vein deposit at one of the mines in Indonesia using the fractal method and compared it to the classical method. In that study, the gold and silver reserves were divided into two zones that were delimited at an elevation of 500 m above sea level. Another study [47] provided an overview of rock mass classes at some underground mines with narrow vein ore types in Indonesia, which were dominated by moderate-to-weak rock. The block model was then created to represent the same conditions.

5.1 The Block Model

Figure 6 explains the uniform dimension block model created from the minimum and maximum ranges of easting, northing, and elevation, respectively, of 100, 100, 35 to 145, 267.5, 102.5. The number of blocks on the x, y, and z axes is 18, 67, and 27, respectively, so the total block model is 32,562. The rock density is set at 2.36 tons/ m^3 , while the gold grade in the block is divided into six zones, namely upper-high, upper-mid, upper-low, lower-high, lower-mid, and lower-low. The upper and lower zones are separated at an elevation of 60 m above sea level, while the high and mid zones and the mid and low zones are separated at a northing of 210 and 150, respectively. The gold grade in each zone was then created using Eq. (28), where the base grade for each zone (upper-high, upper-mid, upper-low, lower-high, lower-mid, lower-low) is 18.8 g/t, 9.4 g/t, 4.7 g/t, 9.9 g/t, 4.9 g/t, and 2.5 g/t. To better represent the real condition, a random number is introduce to randomize the base Additionally, Table 2 shows the economic parameters used in the block's economic calculation.

The same method is applied to the creation of Q' values in the block model. The Q' model is not divided into upper and lower zones but only into high, mid, and low zones that are delimited by the same northing as previously described. Equation (29) explains how the Q' value is created in the

Table 2	Economic parameter	Parameter	Value
		Metal price (\$/gram)	54.8
		Mining cost (\$/ton)	35.8
		Processing cost (\$/ton)	1.6
		Refining cost (\$/ton)	3.9
		Global recovery (%)	80

block model, where the Q' base in the high, mid, and low zones is set to 10.1, 1.1, and 2.7, respectively. Sulistianto et al. [48] determined the rock mass class conditions at one of the underground gold mines in Indonesia, which was used as a basis for the value of 0.5 for factors A and B. Factor C is set to 8 because the assessment in this algorithm is only done on the stope walls that have a vertical orientation.

 $AuGrade! = randomnumber \times Augradebase$ (28)

$$Q' = randomnumber \times Q' base \tag{29}$$

5.2 The Test Methodology

A number of scenarios are used to see the performance of the Mathews analysis in the algorithm that can produce the stope dimension variable in the algorithm. In this validation, three scenarios are used, including the fixed maximum stope dimension, the fixed minimum stope dimension, and the proposed algorithm, which are subsequently referred to as scenarios 1, 2, and 3. The three scenarios were created to explore the optimization potential of stopes with varied shapes due to the variations in rock conditions compared to the commonly practiced optimization based on fixed dimensions. Scenarios 1 and 2 represent optimization with fixed dimensions, while scenario 3 represents stopes with varied. Nevertheless, to enable a comparison between scenarios, the stope height for each scenario is set at 5 m.

In scenario 1, the stope dimension is set according to the maximum dimension allowed based on best geomechanical conditions found in the block model data. The calculations are performed based on the N' value for the best rock condition found in the block model. The largest dimension is determined by calculating the HR value for that rock condition. Because the stope height is fixed, the width (l) and length (w) of the stope can be determined. It is known that

Table 3	validation scenarios	Scenario	Stope dimension $(l \times w \times h)$ (m)
		Scenario 1	7.5×7.5×5
		Scenario 2	$5 \times 5 \times 5$
		Scenario 3	Variable

 Table 4
 Optimization results

Parameter	Scenario 1	Scenario 2	Scenario 3
Number of stopes	195	711	302
Mined tonnage (ton)	238,777	214,161	165,658
Mined metal (grams)	864,736	949,086	856,880
Mined average grade (g/t)	3.62	4.43	5.17
Economic value (\$)	26,283,967	30,640,495	28,700,093

the width and length of the stope based on the rock condition in the block model are 7.5 m. Compared to scenario 1, the minimum stope width and length allowed in scenario 2 is based on operational considerations set at 5 m. This minimum dimension is also applied in scenario 3 as the basis for achieving operational considerations in the optimization phase. The optimization application based on geomechanical considerations is manifested in scenario 3, where the algorithm is given the freedom to determine stope dimensions according to the geological, economic, and geomechanical considerations available in the block model. The scenarios in this case study are seen in Table 3.

To assess the validity of stability analysis application within the stope optimization algorithm, a validation was conducted through back analysis on the final stope walls. Analysis on all four stope walls was carried out by plotting the hydraulic radius (HR) of the stope wall against the stability number (S). The optimization results can be considered valid if all plots of the stope walls fall within the stable zone. Further details are provided in Section 8.

6 Results

Table 4 shows the results of the optimization in the three scenarios, while the results of the stopes that have been optimized are shown in Fig. 7. From Table 4, it can be seen that the integration of the Mathews stability module into the algorithm



Fig. 7 a-c Optimized stopes

in scenario 3 has a level of economy that is competitive with the optimization of the fixed dimension stopes. In general, the varied shape of the stope, following the ore shape, provides an advantage in minimizing mined material waste, as indicated by the highest mined grade among the three scenarios. Furthermore, mining can be considered more efficient. The smaller amount of mined material in scenario 3 indicates that there is less material to be moved for a higher economic value.

In scenario 2, however, the value of the stopes is higher than in scenario 3, which is considered normal for optimization to be carried out on the same cost components in each scenario. As optimization is carried out on the same cost basis across all scenarios, the results will tend towards smaller dimensioned stopes as they can maximize the reserves. In actual case studies, smaller stopes can lead to lower productivity, resulting in relatively smaller economic value due to higher mining costs. The further relationship between the dimensions and the cost of the stopes needs to be established for the algorithm to perform better.

Figure 7 shows the visualization of the optimized stope in all three scenarios. Scenario 1 is unable to maximize the reserves in areas with low grades because the large stope size causes a lot of mined waste material, so the profit from mining ore must compensate for this. This condition is visible in Fig. 7a at low elevations in the southern part, where no stopes are formed in those locations, indicating stopes with negative economic value. In contrast, scenario 2 (Fig. 7b) and scenario 3 (Fig. 7c) showed the opposite results, where the smaller stope dimensions in scenario 2 were able to accommodate ore grade variations better, while the flexibility in dimension selection in scenario 3 had the same effect.



Validation of stope stability was performed on each final stope wall formed in scenario 3. The stability number (S)values for each wall were plotted against the corresponding hydraulic radius (HR) values of the stope wall to determine the stability condition of each wall using Mathews' stability graph. Based on the back analysis plots conducted, all of the optimized stope walls have stable conditions, as indicated by the stability points plotted above the stability line in Fig. 3. In the hanging wall and footwall areas, all stope walls can be considered stable, as seen in Fig. 8 (left). The distribution of the plots tends to be vertical, indicating variations in the rock conditions within the block model, while the tight horizontal distribution indicates consistent stope wall areas. A similar pattern is observed in the distribution of plots for the front wall and back wall (Fig. 8 (right)), suggesting a similar condition in those areas. The tight horizontal distribution also indicates minimal variations in the length or width of the formed stopes, which is commonly observed in narrow deposit formations as utilized in this case study. Further, the application of the Mathews stability analysis to the optimization algorithm was deemed successful, as indicated by the good stope economic values and stable conditions at each stope wall.

8 Conclusions

One of the significant challenges in underground mine planning is determining the stope layout, which involves the location, wall area, and stope size. The stope layout



Fig. 8 Stability analysis for optimized stope wall



dictates the amount of material extracted, the metal content, and grade of minerals extracted, ultimately determining the aggregate economic value of all extracted stopes. Addressing this challenge has been largely done through the development of optimization algorithms, enabling mining engineers to assess various stope layouts more efficiently. However, the involvement of numerous parameters renders optimization algorithms susceptible to suboptimal conditions, wherein the stope layout produced by the optimization process may not be the best solution achievable for a given case study. Among the multitude of parameters involved, rock conditions are one of the dominant parameters considered in determining the stope layout. The study's proposed algorithm tried to combine the steps of stope optimization with stability analysis using the Mathews Stability Chart. This was done so that a more complete design could be made, especially since more field data about the geomechanical properties was available.

Integration is suggested by establishing dimensional constraint at the onset of optimization utilizing Mathews stability analysis. This involves incorporating geomechanical data, such as rock mass classification Q', factor A, factor B, and factor C. The outcomes demonstrate the effectiveness of this approach, evident in the optimization outcomes that yield superior economic value compared to employing fixed dimensions. Furthermore, the stability of the optimized stope walls affirms the effectiveness of stability analysis within the optimization algorithm, ensuring the project's financial feasibility. The results of optimization by this algorithm could serve as preliminary guidance for mine planners during the feasibility evaluation phase.

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Declarations

Competing Interests The authors declare no competing interests.

References

- Mawdesley C, Trueman R, Whiten WJ (2001) Extending the Mathews stability graph for open-stope design. Min Technol 110:27–39. https://doi.org/10.1179/mnt.2001.110.1.27
- Lu A, Yin C, Zhang N (2019) Analytic stress solutions for a lined circular tunnel under frictional slip contact conditions. Eur J Mech A Solids 75:10–20. https://doi.org/10.1016/J.EUROMECHSOL. 2019.01.008
- Vitali OPM, Celestino TB, Bobet A (2019) Shallow tunnels misaligned with geostatic principal stress directions: analytical solution and 3d face effects. Tunn Undergr Space Technol 89:268– 283. https://doi.org/10.1016/J.TUST.2019.04.006
- Napa-García GF, Câmara TR, and Navarro Torres VF (2019) Optimization of room-and-pillar dimensions using automated numerical models. Int J Min Sci Technol 29: https://doi.org/10.1016/J. IJMST.2019.02.003.
- Abdellah WR, Ahmed HM, Hefni MA (2019) Numerical modelling of staged stope extraction in a tabular steeply dipping deposit. Geomech Geoeng 14:41–51. https://doi.org/10.1080/17486025. 2018.1508856
- Mensah T (2023) A binary integer linear programming model for optimizing a binary integer linear programming model for optimizing underground stope layout underground stope layout. https://scholarsmine.mst.edu/masters_theses/8176
- Lowson AR, Bieniawski ZT (2013) Critical assessment of RMR based tunnel design practices: a practical engineer's approach. SME
- Kang Z et al (2019) Optimization calculation of stope structure parameters based on Mathews stabilization graph method. J Vibroeng 21:1227–1239
- 9. Wang D et al (2021) Stope stability assessment by the Mathews-Potvin method: a case-study of open stoping in salt rock mass under conditions of secondary stress field. IOP Publishing. https://doi.org/10.1088/1755-1315/684/1/012011
- Liu H, Zhao Y, Zhang P, Liu F, Yang T (2021) Stope structure evaluation based on the damage model driven by microseismic data and Mathews stability diagram method in Xiadian gold mine. Geomat Nat Haz Risk 12:1616–1637. https://doi.org/10. 1080/19475705.2021.1941308
- Janiszewski M, Pontow S, Rinne M (2021) Industry survey on the current state of stope design methods in the underground mining sector. Energies (Basel) 15:240. https://doi.org/10.3390/ en15010240
- Nikbin V, Ataee-pour M, Shahriar K, Pourrahimian Y, MirHassani SA (2018) Stope boundary optimization: a mathematical model and efficient heuristics. Resour Policy 62:515–526. https://doi.org/10.1016/J.RESOURPOL.2018.10.007
- Basiri Z (2018) Stopes layout and production scheduling optimization in sublevel stoping mining. Dissertation, University of Alberta
- 14. Nelis G, Gamache M, Marcotte D, Bai X (2016) Stope optimization with vertical convexity constraints. Optim Eng 17:813–832. https://doi.org/10.1007/s11081-016-9321-6
- 15. Wilson B (2020) Heuristic stochastic stope layout optimization. University of Alberta, Thesis
- Furtado e Faria M, Dimitrakopoulos R, and Lopes Pinto CL (2022) Integrated stochastic optimization of stope design and long-term underground mine production scheduling. Resources Policy 78: https://doi.org/10.1016/j.resourpol.2022.102918.
- Furtado e Faria MA, Dimitrakopoulos R, Pinto C (2022) Stochastic stope design optimisation under grade uncertainty and dynamic development costs. Int J Min Reclam Environ 36:81– 103. https://doi.org/10.1080/17480930.2021.1968707

- Sari YA, Kumral M (2021) Sublevel stope layout planning through a greedy heuristic approach based on dynamic programming. J Oper Res Soc 72:554–563. https://doi.org/10.1080/ 01605682.2019.1700179
- Tolouei K, Moosavi E, Tabrizi AHB, Afzal P, Bazzazi AA (2021) An optimisation approach for uncertainty-based longterm production scheduling in open-pit mines using meta-heuristic algorithms. Int J Min Reclam Environ 35:115–140. https:// doi.org/10.1080/17480930.2020.1773119
- Lamghari A, Dimitrakopoulos R (2020) Hyper-heuristic approaches for strategic mine planning under uncertainty. Comput Oper Res 115:104590. https://doi.org/10.1016/j.cor.2018.11. 010
- 21. Ovanic J, Young DS (1995) Economic optimisation of stope geometry using separable programming with special branch and bound techniques. McGill University
- Riddle JM (1977) Dynamic programming solution of a blockcaving mine layout. 767–780. https://www.onemine.org/docum ents/a-dynamic-programming-solution-of-a-block-caving-minelayout Accessed: Jan. 26, 2023
- Deraisme J, De Fouquet C, Fraisse H (1984) Geostatistical orebody model computer optimization of profits from different underground mining methods. 583–590
- 24. Cheimanoff NM, Deliac EP, Mallet JL (1989) GEOCAD: an alternative cad and artificial intelligence tool that helps moving from geological resources to mineable reserves. Publ by Soc of Mining Engineers of AIME
- 25. Alford C (1995) Optimisation in underground mine design.
- 26. Cawrse I (2001) Multiple pass floating stope process.
- 27. Ataee-Pour M (2004) Optimisation of stope limits using a heuristic approach. Inst Mining Metall Trans Sect A: Min Technol 113: https://doi.org/10.1179/037178404225004959.
- Topal E, Sens J (2010) A new algorithm for stope boundary optimization. J Coal Sci Eng 16:113–119. https://doi.org/10.1007/ s12404-010-0201-y
- Sandanayake DSS, Topal E, Ali Asad MW (2015) A heuristic approach to optimal design of an underground mine stope layout. Appl Soft Comput 30:595–603. https://doi.org/10.1016/J.ASOC. 2015.01.060
- Bai X, Marcotte D, Simon R (2013) Underground stope optimization with network flow method. Comput Geosci 52:361–371. https://doi.org/10.1016/j.cageo.2012.10.019
- Sandanayake DSS, Topal E, Asad MWA (2015) Designing an optimal stope layout for underground mining based on a heuristic algorithm. Int J Min Sci Technol 25:767–772. https://doi.org/10. 1016/j.ijmst.2015.07.011
- 32. Sandanayake DSS (2014) Stope boundary optimisation in underground mining based on a heuristic approach. Dissertation, Curtin University
- Sari YA, Kumral M (2019) A planning approach for polymetallic mines using a sublevel stoping technique with pillars and ultimate stope limits. Eng Optim 52:932–944. https://doi.org/10.1080/ 0305215X.2019.1624739
- Sari YA, Kumral M (2021) Clustering-based iterative approach to stope layout optimization for sublevel stoping. J South Afr Inst Min Metall 121:97–106. https://doi.org/10.17159/2411-9717/ 1237/2021

- Kumral M, Sari YA (2020) Underground mine planning for stopebased methods. AIP Publishing LLC 10(1063/5):0006787
- Villalba Matamoros ME, Kumral M (2017) Heuristic stope layout optimisation accounting for variable stope dimensions and dilution management. Int J Min Miner Eng 8:1–18. https://doi.org/10. 1504/IJMME.2017.082680
- Hou J, Xu C, Dowd PA, Li G (2019) Integrated optimisation of stope boundary and access layout for underground mining operations. Min Technol 128:193–205. https://doi.org/10.1080/25726 668.2019.1603920
- V Nikbin E Mardaneh M Waqar A Asad E Topal 2021 Pattern search method for accelerating stope boundary optimization problem in underground mining operations https://doi.org/10.1080/ 0305215X.2021.1932869
- Esmaeili A, Hamidi JK, Mousavi A (2023) Determination of sublevel stoping layout using a network flow algorithm and the MRMR classification system. Resour Policy 80:103265. https:// doi.org/10.1016/J.RESOURPOL.2022.103265
- Laubscher DH (1990) A geomechanics classification system for the rating of rock mass in mine design. J South Afr Inst Min Metall 90:257–273
- 41. Potvin Y, Hadjigeorgiou J (2001) The stability graph method. Underground Mining Methods 66:513–520
- 42. Mathews, K. E. Hoek, D. C. Wyllie, and S. B. V. Stewart (1980) Prediction of stable excavation spans for mining at depths below 1000 metres in hard rock. https://www.scirp.org/(S(i43dyn45te exjx455qlt3d2q))/reference/ReferencesPapers.aspx?ReferenceID= 1053122 Accessed: Mar. 02, 2020
- 43. Potvin Y (1988) Empirical open stope design in Canada. University of British Columbia
- 44. Nickson SD (1992) Cable support guidelines for hard rock mine operations. University of British Columbia
- Nhleko A, Tholana T, Neingo P (2018) A review of underground stope boundary optimization algorithms. Resour Policy 56:59–69. https://doi.org/10.1016/J.RESOURPOL.2017.12.004
- Prasetyo E (2010) Fractal model and classical block model in ore reserve estimation: a comparison. Riset Geologi Dan Pertambangan 20:119–130
- Purwanto SH, Sasaoka T, Wattimena RK, Matsui K, Matsui K (2013) Influence of stope design on stability of hanging wall decline in Cibaliung underground gold mine. Int J Geosci 04:1–8. https://doi.org/10.4236/ijg.2013.410A001
- Budi S, Wattimena RK, Ardianto A, Matsui K (2009) Determination of stope geometry in jointed rock mass at Pongkor underground gold mine. Int J JCRM 5:63–68. https://doi.org/10.11187/ ijjcrm.5.63

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Integration of Stability Factor A, B, and C on Mathews Stability Graph

by Danu Putra FTKE

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Integrating Mathews Stability Chart into the Stope Layout Determination Algorithm

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Abstract

Stepe layout optimization is an important 12 new of underground mining that maximizes the accountic value of the project while taking mining limits into account. The large number of parameters and constraints trakes it difficult to obtain the optimum condition. Several algorithms have been created to address these problems using a variety of methods. However, this circulating method has not explicitly included single dimension stability analysis, resulting in a solution that in rul stability-proven, which can result in a suboptimal obtain. This study integrates the Mathews subfility graph into the tops optimization algorithm so that the optimized stope layout considers stability conditions directly through an assessment of the available geomechanical data within the block model. The proposed algorithm is validated through a case study of a synthetic block model created by considering variations in grafa and the geomechanical conditions of the top: Furthermore, several secturity of stope sizes that remain fixed. A more detailed assessment is also conducted on each flual stope layout will in ensure the successful application results show that all walls in the intrust stope layout fail into the stable condition. Also, the proposed algorithm is also capable of maintaining the project's contourie tay of a synthetic biock model. The proposed algorithm through hack analysis on the Mathews withing graph. The optimization ensure is also conducted on each flual stope layout wall in ensure the successful application results show that all walls in the intrust stope layout fail into the stable condition. Also, the proposed algorithm is also capable of maintaining the project's construct vise. Elitinately, the proposed algorithm can be deeme 11 plucation and successful properties of the steps.

Reywords Stope optimization - Heuristic algorithm - Mathews Stability Chart - Stope layout - Underground mine

Anotopi

1 Introduction

Underground stope stability analysis methods have been widely developed using various techniques such as empirical [1], analytical [2, 3], and numerical [4, 5] in line with the increasing complexity of rock conditions with deeper mining. The increasing complexity of underground mines

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creates challenges regarding stope design. Poorer tock conditions result in smaller stopes, which then limit reserve. while better rock conditions tend to accommodate higger stope dimensions [6]. To accommodate the complexity and further simplify the stability analysis. Critical Span Graphs [7] and Mathews Stability Chart [8-10] are two of the empirical approaches that are still widely used as industry standards. These approaches could help steginestes determine the stability of stope designs faster, thus making the generation of specific stope designs in certain geomechanical conditions possible. While varying stope dimensions to their specific geomechanical proparties could have significant impacts on mine reserves, thus creating more value in mino projects. stone design is often limited in a conservative way, such in when poorer geomechanical data is chosen as the basis for the stope dimension. Furthermore, research integrating stability topics with stope optimization is still limited while such studies are imperative [11].

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Many stope optimization algorithms have been developed to assist engineers in solving optimization problems [12-14]. Various optimization techniques, such as stochastic [15-17], exact algorithms, and heuristic algorithms [18-20]. have been used to ensure that the stone optimization results in maximum NPV with stope dimensions as constraints. The exact algorithm is formed from a mathematical model, ensuring the best solution is obtained. Some algorithms falling into this category include Branch and Bound [21]. Dynamic Programming [22], and Downstream Genstatistical Approaches [23]. Dynamic Programming stope optimization was introduced by Riddle [22] in a block-caving case study. which also has its weaknesses due to limitations in its applicution to that method. Demine et al. [23] introduced the downstream geostatistical approach to determine parts of the ore body with the best economics. However, the application of this algorithm is finited to cut-and-fill or sublevel stoping methods. The solutions generated by this algorithm carnot yot be considered optimal as they have not been proven with practical mining designs. Ovanic and Young [21] further developed the Branch and Bound almovithm applied to integer programming. Generally, Integer programming requires considerable assources for problem-solving, making it often unfeasible for large case studies. The integration with the Branch and Bound algorithm allows problems to be broken down into smaller ones, making the problem-solving process more efficient. Nevertheless, the effectiveness of solving problems in large case studies remains a weokness, so this algorithm has not been applied beyond one-dimensional case studies.

Contrary to exact algorithms, heuristic algorithms do not focus on mathematical models: flux, the solutions they generate do not fully achieve the global optimum but are close enough to the global optimum. Some algorithms falling into this category include Octree Division [24]. Ploating Stope [25], Multiple Pass Floating Stope [26], Maximum Neighborhood [27], Topal and Sens [28], and Sandamyake [29]. The Octroe Division algorithm introduced by Cheimanoff et al. [24] is capable of working in three-dimensional case studies where the "optimum" part of the block model is determined based on mining constraints and economics. In practice, this algorithm approaches the optimal condition in producing a 3D store knoot. However, the structure of the aborithm, which allows waste blocks to enter the final stope fayout; reduces the economic value of the final stope fayout, Henze, the optimal solution has not been achieved yet. The next development in stope algorithms, which is quite applicable and adopted in commercial software, is the Floating Stope by Alford [25]. The approach used is similar to other algorithms in open-pit case studies. 15 h as the Floating Cone. Similar to the Floating Cone, one advantage of this algorithm is its simplicity, where a stope of predetermined dimensions is floated on the block model, and an

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assessment of the stope is conducted to determine its economic feasibility. However, a weakness of this algorithm lies in not considering the important concept of overlapping stopes. Overlapping stope solutions result in double-counted reserves, leading to increased economic feasibility. Adjustments have to be made to ensure that the mined material truly remesents the actual mine and its economic value. The need for manual intervention in this algorithm means that the solution from the Ploating Stope algorithm cannot yet be considered optimal. To address this weakness, Course [26] developed the Midtiple Pass Floating Stope, providing additional information to engineers and moking the assessment of the Floating Stone output easier. However, the main weakness of the overlapping stope concept has not been resolved, causing this algorit 14 to still be unable to produce on optimal solution. Still based on the principle of the Floating Stope, the Maximum Neighbothood algorithm was developed by Atase-Pour [27]. A more detailed approach, aggregating blocks into ctope shapes and, in the process, eliminating blocks with negative economic value, makes this approach better. However, the solutions penetated are highly dependent on the initial location of the optimization iterations, causing this algorithm to not yet produce an optimal solution. Later, the hearistic approach developed by Topal and Sens [28] changes geological blocks into economic blocks of uniform size and then forms stopes of specitic dimensions in each block model, assessing the stope attributes to see their feasibility. One breakhrough of this approach is the final output of the stope in three dimensions. The structure of the algorithm that sequentially eliminates sets of stopes is a weakness of this approach, making the optimal stope layout not necessarily achievable. Bai [30] developed a hearistic algorithm applied to the sublevel mining method. The limitation of this approach lies in the mining method's conditions and its application, which can only be applied to until ore bodies. Finally, Sandanayake [29] developed a heuristic algorithm by modifying the Hosting Stope, where first, the block economic value (BEV) is determined by calculating all oconomic and goological componexts within the block. Then, stopes of specific sizes are Boated within the block model, while the economic value of the stope is calculated based on the camulative BEV values entering the stope. Elimination is then carried out on stopes with negative economic value, while sets are formed on stope 12 th positive values that do not overlap. The set of stopes with the best deeponic value is chosen as the best solution. However, calculating BEV at the beginning of optimization is one of the weaknesses of this algorithm because remonic parameters are independent of mining scenarios, thus the possibility of hidden positive economic value stopes not being further assessed.

The early slope optimization algorithm presented has a common way to express stable slope dimension. The slope,

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as a minesble area, is typically simplified into a box-staped dimension with floating width, length, and height [26, 31, 32]. To address practical requirements, dimensional constraints were implemented to ensure that the optimization outcomes met gentechnical and technical conditions (33, 34]. Dimension considerations in optimization are focused on two approaches: fixed dimensions [29, 31] and variable dimensions [35-38]. Fixed dimensions impose uniform predefined stope dimensions at the initial optimization stage. This constraint limits the algorithm's flexibility in selecting the best stope due to the predetermined size set by the user at the start of optimization. Meanwhile, variable dimensions were applied by setting the maximum and minimum dimension constraints allowed for the stope favour. By providing moximum and minimum dimension constraints, the optimixed stope layout fulfills both geomechanical and operational considerations. However, both approaches require users to determine the generally allowable stope size in each optimization domain. Furthermore, variations in rock conditions are not directly considered in the optimization algorithm.

The use of stability analysis in stope optimization algorithm is still limited, used separately from optimization steps, where the most pessimistic geomechanical data is usually used as the basis for determining mining design in a wider area. Limited geomechanical data provides a large amount of uncertainty and eliminates economic gotential, as some areas may have marginal value when mined with different stope dimensions. The latest study that adopted stability analysis in optimization algorithms was conducted by Esmacili et al. [39] by applying stability analysis to the Caving Graph [40] as mining constraints combined with a network flow algorithm. The algorithm was successfully applied to the sublevel caving mining method with listited block numbers. As stope stability analysis methods are widely available, the potential of integrating stability analysis with currently available optimization algorithms is significant. The advantage of this methodology lies in the ability of the algorithm to read and analyze geomedianical data that is already quantitatively available to create stope dimension recommendations. This study aims to integrate Mathews stability analysis [41] into a mining optimization algorithm [32].

2 Proposed Algorithm

Stope designs that represent variations of rock conditions are needed to maximize the project values. In this study, the stope optimization algorithm [31] was modified by adding a stage of stope dimension recommendation based on the Mathews stability graph [43], making the overall algorithm stages as shown in Fig. 1. The approach was carried out by calculating the stability number (N) based on the geomechanical parameters available in the block model, which include factor A, factor B, factor C, and Q' value, N assobers are generated throughout iteration based on the available stopes wills that are constrained within the ore body. Further, the maximum allowable hydraulic radius is determined to limit the maximum stope dimension in each block location in the block model. Optimization was then carried out on a similar basis, but with the addition of the geomechanical constraint that was newly proposed.

In this study, the application of the proposed algorithm was limited to the open-stope or sublevel method for metal mines, as the Mathews Stability Chart suggested. Further, consideration of the angle, theckness, fush, and aquifer of the one body was considered in the parameters utilized in the Mathews Stability Chart that were already presented in the block model in the form of *factor* A, B, C, and Q' value, while wope will orientation was limited to vertical as the base of the algorithm was limited to [31].

As for the mathematical models become very complex, Table 1 summarizes the notations, parameters, and decision variables that were used for the subsequent sections.

2.1 Objective Function

The objective function of this study is to maximize the stope recommic value by accumulating the block economic value inside the optimum stopes. This was done by utilizing Eq. (1) In order to determine the stopes construct value, two main parameters were used: the geological parameter, including metal grade, and the economic parameter, including metal price and cost components. The block value is calculated in some sequences. First, block tuninge was determined by block lengths and week density via Eq. (2). After the tuningy values of the blocks are known, the occorrite value of the block is calculated using Eq. (3) by applying reconcrising cost, and selling cost.

MAX	$\sum v_r \approx \text{Augur}$	(1)

$\sigma = Ha \times La \times Wa \times da$	(2

 $\mathbf{r}_{c} = \left[\left(\mathbf{p} - \mathbf{r}_{c} \right) \otimes \mathbf{y}_{c} \otimes \mathbf{y} - \left(\mathbf{r}_{c} + \mathbf{z}_{c} \right) \right] \otimes \mathbf{t}_{c} \qquad (3)$

2.2 Stope Height Constraint

The stope height constraint limits the maximum optimized stope height by ensuring that the cumulative height of mining blocks on the c-axis ($m_{2,L}$) does not exceed the allowable stope height. Stope height is determined by considering

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the mining method that is applied in the area and set by the user. Allowing one axis to be fixed decrutises the complexity of the algorithm as it will only optimize the stope length and spen. However, fall consideration needs to be given by the user, as the stope height will also dictate the mining level. A shorter stope will generate many levels and further impact the need for mine access while also creating the opportumity to do selective mining. On the contrary, a higher stope will generate fewer levels, but the production rate could be higher, further impacting the production cost. This condition is displayed in Fig. 2 where the stope's origin, positioned at the lowest elevat 1 of the block model (marked by the green-colored box), is the determining block for the stope's height constraint (ezs,), which also defines the number and location of levels. Nevertheless, the final layout will be driven by the rock conditions, as this constraint only enforces one of the three axes.

In the proposed algorithm, the stope shape is controlled via block quantity relative to its axis. Thus, conversion from allowable steps bright to allowable mining block is needed. Equation (4) implies that the maximum stope bright is converted by dividing the maximum height by the block height.

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The calculation was possible because of the regularity of the block size.

 $\mu_{2N_{c,1,0}} = Hman/Ho$

(4)

2.3 Maximum and Minimum Width Constraint

The maximum and minimum constraints limit the stope size during the optimization process, ensuring that the optimal stope meets operational and geomechanical enterial the minimum operational width defined by the user (Woiva). The stope's width should accommodate the explorment ion operating in that area. The use of anechanical equipment tends to require a larger minimum width for the stope compared to inditional mining. In some cases within narrow veins, the equipment width may conflict with the vein width, necessitating a wider stope to compose at the machanized mining activities in that location. Further impacting the increase in planned dilution, thus reducing the economic feasibility of the stope.

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Symbol	Descriptions
Netscheite	
1	linhox position of a
1	latix position of y
5	Infers position of z
Parameter	12. Of the Second Constants
Ar.	Factors A Mathews stability graphs in block
Re	Factory & Mathews stability graphs in block
0	Factors C Mattews subility graphs in block
Úr.	Rock density
87	Metal grade to block
ev	Metal grade in stops
Abraco	Maximum supplicing a
the	Block length
inte.	Hydraphic radius on the 3-th wall of the stope
AMR22	Hydrophy malase on the 2-th wall of the stone
ANR SP	Hedroalic radius on the 3-th wall of the stope.
118.4	Hydraulic matters on the 4-41 wall of the store
Louis	Maximum length of same determined by page
Louise	Minimum length of some descripted by east
La	Rinsk length
-	Metal weight in black
	Mentil waiete in more
1111	Wassaher on the 5-th scall of the store
A2+	Wassenbeicten (he 2 th scall of the steps
ABr	Wasanbey on the 3-th wall of the stope
NA	Seesier on the 4-th wall of the secs
W	Namber of blocks in 2-directing
	Namber of blacks in volimentian
1	Number of blacks in softreetion
1110	N-blocks towards a an allower on the new
arity.	N-blocks towards y an allowed on the score
	N March Strength, 7 and all march on the state
0r	O'value Dousterrie Mark
	One transmer in block
0	Our comments in success
	Store company value
We .	Kind with
Warker	Maleran with of store determined becaute
Wanti	Maximum width of store determined to the ner-
Decision wreakles	And the second of the second s
the second second second	The first one block
Time	The for our block in stress
	Desiries are been been dealers

Differing from the minimum width (Weeks), the minimum stope length (*Leoise*) is typically determined based on the length of the stope advancement, where its width is no smaller than the minimum stoge advancement length. This constraint ensures that no stope design is created smaller than the stope advance length.

A significant factor affecting the maximum stope width and length (Wwarva and Lenara) is geomechanics. Solid nuck, limited water presence, and favorable strong conditions are indicators of favorable rock conditions where stope sizes can generally be larger to meet production oreds. Furthermore, in the design aspect, the orientation of the tracture and stope walk can be a determining factor for stability/sofety in stope design. Mithews [42] proposed an empirical approach applicable to open stopes or sublevel stoping, where the hydenalic radius and stability tope dimensions. The application and megation of the stope dimensions. The application and megation of the geomechanical constraint model into the stope dimension constraints are explained in more detail in Section 4.

Geomechanical constraints are established by ensuring that the stopes have dimensions smaller than the allowed hydrodic radius at the location where the stopes will be formed. Meanwhile, operational constraints drauer that the dimensions of the stopes are larger than the minimum allowed dimensions at a block model location. Both of these constraints are combined in a smilled constraint that regulates the maximum and minimum dimensions along the *x*-axis ($mx_{1,2}$), *x*-axis ($mx_{1,2}$), and *z*-axis ($mx_{1,2}$), *x*-axis ($mx_{1,2}$), and *z*-axis ($mx_{2,2}$). The stope size is limited by Eq. (5) to (6), which add up the indices of the mined blocks (mgs) in the more layout and compare them to the stope size limits. Equations (5), (6), and (7), respectively, operate on the *x*-axis, *y*-axis, and *z*-axis.



The application of these equations serves as a constraint in the stepe optimization phase, as depicted in Fig. 3. The green-colored blocks depict the block origin's position where the stope dimension constraints are applied, while the doubed red lines represent the boundary of the stope layout's location with the application of stope dimension constraints. With the integrated application of geomechanteral considerations in the stope dimension constraint at each stope location, the stope dimensions can be deemed representative as they meet the mck conditions.

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3 Maximum Width and Span by Mathews Stability Chart

3.1 Mathews Stability Chart

Mathews [42] introduced a stability graph based on 26 cases collected from open-stope underground mining. This data was later supplemented and recalibrated by Potvin [43]; which became widely used in the industry as the basis for 10 se planning that considers rock genmachanics conditions. The stability graph represents a plot of the stability number (N') against the shape factor (5) or also known as the hydron-8 radius (HR). The calculation of N is done by considering Rock Quality Designation (RQD) joint set number (J_1) joint roughness sumber (J_i) , joint alteration number (J_i) , stress factor (A), joint orientation factor (B), and gravity factor (C) through Eq. (9). Meanwhile, HR is generally the ratio between the area and the perimeter, which is determined based on the length (L) and width (W) of the stope 10, as shown in Eq. (10). Both of these variable results are plotted on the stability graph to determine the stability condition of the wall through three zones depicted on the graph: stable, unstable, and cave, as seen in Fig. 5.

$$Nr = \left(\frac{RQD}{J_{\mu}}\right) \times \left(\frac{J_{\tau}}{J_{\mu}}\right) \times A \times B \times C$$

(91

(10)

$$HR = \frac{W \times L}{2 \times (W \times L)}$$

This study employs the stability graph developed by Potvin [43] and Nickson [44] as a stability analysis tool within



Fig.5 Force-modified Matteries earliery graphs [43]

the optimization algorithm. Equation (11) represents the boundary between the stable and routable areas on the stability graph which was statistically calculated by Nickoon [44] based on 175 case studies of stope stability in Potvin (43). The stope wall dimensions allowed fall within the areas above this houridary line. Through the use of this stability graph, it is also assumed that the stopes used in this algotilhm are unsupported.

 $HR = 10^{10.225 + 0.100 \text{kgpcs}}$

3.2 Mathews Stability Chart Application in Stope Dimensional Constraint

In this study, improvements to the existing stope optimization algorithm were made by incorporating stability analysis using Mathews Stability Chart into the algorithm as dimensional constraints. Thus, the proposed algorithm has the ability to directly address seek conditions. This was done by assessing the Mathews attribute data provided in the block model. The analysis is conducted at block locations by iterating steps as follows:

- Assessing the maximum of each stope wall domain by looking for the ore domain.
- Calculate the N stability reserver based on Q' value, factor A, factor B, and factor C within the will domain.
- Determine the stable condition for the wall domain by correlating the hydraulic radies and the N stability now-
- brr. 4. Assessing a smaller domain until a stable condition is
- thet
 Determine the allowable stope wall dimensions by during the lowest hydraulic conditions between each

Revi The algorithm steps are handled by several equations, as follows Eqs. (12) and (13) are used for the first step, to determine which part of the tock is the ore body, so that subsequent iterations of the equation will be limited to the controlled by the i, j, and & indices of the block. The difference between the two equations lies in the orientation in which each equation is applied. Equation (12) is utilized on the stope wall oriented towards the east, while Eq. (13) is applied on the stope wall oriented towards the north. The calculation of the N stability number in the second step is performed by utilizing Eqs. (14) to (17). Each of the equations represents the different calculations performed for the four staffs. As seen in the equations, all four equations have different block location indices, representing calculations for block data in different domains of the stope walls. The N stability number for each of the stope

wall domains is then used to calculate the allowable indom/in

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The selection of the lowest hydraulic radius is then performed on opposing walls to ensure that the lowest value to be used is indicated by Eqs. (22) and (23). The allowable length for stope walls is then determined based on the hydraulic

matter of the corresponding wall through Eqs. (24) and (25). In the last stage, Eqs. (26) and (27) are making sure that the allowable length for the stope wall has already met operational constraints.





1.

2

8

4

5.

6.

previously developed houristic algorithms is Sandanayake [29, 31]. Sandanayake [29] introduces a hearistic algorithm for stope optimization with the following general steps:

Initialization of all data, parameters, and variables

Stope formation through mining block aggregation

Extraction of subsets with accesorie values greater than 0

Update of stope attributes based on mined blocks

Identification of overlapping stopes

4 Optimization

Nhelko A [45] emphasizes the limitations of exact algorithm application in large-scale cases such as stops optimication. Although the resulting solutions may achieve the global optimum, the problem-solving time typically increases exponentially with problem complexity, making this algorithm category infeasible for large-scale cases. Heuristic algorithms are commonly employed solutions for addressing couplex problems like stope optimization,



The algorithm was modified by incorporating Mathews' stability considerations into the stope dimension constraints (nyr, nar, nyr). The varying dimension constraints are hold by each block within the block model in line with the geomechanical conditions of that block. Hence, the stope dimensions will vary according to the geomechanical conditions at that location. These constraint applications are implemented in the initial stage of stope constion. The softsequent stope master relevant, when (2) --symptopic given are constrained at this stope, set with the highest economic value is selected.

5 Case Study

To test the validity of the algorithm proposed in this study, a block model was created by considering several case studies of underground gold mines in Indonesia. Practyc et al. [46] modeled a gold vein deposit at one of the mines in Indone sta using the fractal method and composed it to the classical method. In that study, the gold and silve@eserves were divided into two zeries that were delimited at an elevation of 500 m above sea level. Another study [47] provided an overview of rode mass classes at some underground mines with narrow wein out types in Indonesia, which were dominated by moderate-to-weak rock. The block model was than created to represent the same conditions.

5.1 The Block Model

Figure & explains the uniform dimension block model created from the minimum and maximum ranges of easting. northing, and elevation, respectively, of 100, 100, 35 to 145, 267.5, 102.5. The number of blocks on the r, y, and g axes is 18, 67, and 27, respectively, so the total block model is 32,562. The seek density is set at 2.36 tons/m³, while the gold grade in the block is divided into six zones, namely upper-high, upper-mid, upper-low, lower-high, lower-m 9 and lower low. The upper and lower zones are separated at an elevation of 60 m above sea level, while the high and mid zones and the mid and low somes are separated at a northing of 210 and 150, respectively. The gold grade in each zone was then created using Eq. (28), where the base grade for each zone (upper-high, upper-mid, upper-low, lower-high, lower mid, lower-low) is 18.8 g/t, 9.4 g/t, 4.7 g/t, 9.9 g/t, 4.9 ph, and 2.5 g/r. To better represent the real condition, a random number is introduce to randomize the base Additionally, Table 2 shows the economic parameters used in the block's economic calculation.

The same method is applied to the creation of Q' values in the block model. The Q' model is not divided into apper and lower zeroes but only intrihigh, mid, and low zeroes that are definited by the same northing as previously described. Equation (29) explains how the Q' value is created in the

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Pinonn	Value
Motal price (Signes)	34,8
Mining cost (Mort)	33.8
Processing cost (Schore)	1.6
Refining our difford	3.9
Global recovery (%)	RD

(29)

block model, where the Q² base in the high, mid, and low romes is set to 10.1, 1.1, and 2.7, respectively. Sullistanto et al. [48] determined the rock mass class conditions at our of the underground gold mines in Indonesia, which was used as a basis for the value of 0.5 for factors A and B. Fachor C is set to 8 because the assessment in this algorithm is only done on the stope walls that have a vertical orientation.

AnGrade! = rundommuniber x Augradebase (28)

 $Qt = nandomnosher \times Qtheor.$

5.2 The Test Methodology

Table 2. Scorewise purpractor

A number of accmanics are used to see the performance of the Mathews analysis in the algorithm that can produce the stope dimension variable in the algorithm. In this validation, three scenarios are used, including the fixed maximum stope dimension, the fixed minimum stope dimension, and the proposed algorithm, which on subsequently referred to as scenarios 1, 2, and 3. The three scenarios were created to explore the optimization potential of stopes with varied shapes due to the variations in trock conditions compared to the commonly practiced optimization based on fixed dimensions. Scenarios 1 and 2 represent optimization with fixed dimensions, while scenario 3 represent supper with taried. Nevertheless, to enable a comparison between scenarios, the stope height fixe each scenario is set at 5 m.

In scenario I, the stope dimension is set according to the maximum dimension allowed based on bost geomechanical conditions found in the block model data. The calculations are performed based on the N value for the best rock condition found in the block model. The largest dimension is determined by calculating the *HR* value for that rock condition. Because the stope beight is fixed, the width (I) and length (w) of the stope can be determined. It is known that

Table 3. Validation scenarios

Szenarie Skope Alexen una (hi scrick) (m)

Securio 1 73×75×3 Securio 2 3×3×3 Securio 3 Variable

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able 4. Optimization middle			
Parameter	Scimanio I	Screene 2	Scenario 3
Number of stopes	145	TIL	362
Mised tempe (100)	138.777	234,161	165,638
Minestractal (grama)	864.130	141.085	829.880
Ministeverage gode (g/)	1.42	4.42	5.17
Economic value (5)	16,283,967	10.640.498	18,760,093

the width and length of the stope based on the rock condi-

tion in the block model are 7.5 m. Compared to scenario 1,

for achieving operational considerations in the optimization

phase. The optimization application based on geomechanical

considerations is manifested in scenario 3, where the algorithm is given the freedom to determine stope dimensions according to the geological, economic, and promechanical posiderations available in the block model. The scenarios in this case study are seen in Table 3.

To assess the validity of stability analysis application within the stope optimization algorithm, a validation was runcheted through back analysis on the final stope walls. Analysis on all four stope walls use carried out by plotting the hydraulic radius (*IRb*) of the stope wall against the stability member (3). The optimization results can be considered valid if all plots of the stope walls fall within the stable some. Further details are provided in Section 8.

the minimum stope width and length allowed in scenario 2 is based on operational considerations set at 5 m. This minimum dimension is also applied in scenario 3 as the basis

Table 4 shows the results of the optimination in the three scenarios, while the results of the stopes that have been optimized are shown in Fig. 7. From Table 4, 6 can be seen that the integration of the Mathows stability module into the algorithm



in scenario 3 has a level of economy that is competitive with the optimization of the fixed dimension stopes. In general, the varied shape of the stope, following the ore shape, provides an advantage in minimizing mixed margerial waste, as indicated by the highest mixed grade among the three scenarios. Furthermose, mixing can be considered more efficient. The smaller amount of mixed material in scenario 3 indicates that there is less material to be moved for a higher economic value.

In scenario 2, however, the value of the stopes is higher than in scenario 3, which is considered normal for optimization to be carried out on the same cost components in each scenario. As optimization is carried out on the same cost basis across all scenarios, the results will tend towards smaller dimensioned stopes as they can maximize the reserves. In actual case studies, smaller stopes can lead to lower productivity, resulting in relatively smaller economic value the to higher mining costs. The farther relationship between the dimensions and the cost of the stopes needs to be established for the algorithm to perform better.

Figure 7 shows the visualization of the optimized stope in all three scenarios. Scenario 1 is unable to maximize the concreasing areas with low grades because the large stope size causes a lot of mined wate material, so the profit from mining ore must compensate for this. This condition is visible in Fig. 7a at low elevations in the southern part, where no stopes are formed in these locations, indicating stopes with negative economic value. In contrast, scenario 2 (Fig. 7b) and scenario 3 (Fig. 7c) showed the opposite results, obser the smaller stope dimensions in scenario 2 were able to accommodate one grade variations better, while the flexibility in dimension selection in scenario 3 had the same effect.

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7 Stability Confirmation with Mathews

Validation of stope stability was performed on each final stope wall formed in scenario 3. The stability number (5) values for each wall were plotted against the corresponding hydraulic radius (HR) values of the stope wall to determine the stability condition of each wall using Mathews' stability graph. Based on the back analysis plots conducted, all of the optimized stope walls have stable conditions, as indicated by the stability points plotted above the stability line in Fig. 3. In the banging wall and footwall areas, all stope walls can be considered stable, as seen in Fig. 8 (left). The distribution of the plots tends to be vertical, indicating variations in the rock conditions within the block model, while the tight borizontal distribution indicates consistent wope wall areas. A similar pattern is observed in the distribution of plots for the front wall and back wall (Fig. 8 (right)), suggesting a similar condition in those areas. The tight horizontal distribution also indicates minimal variations in the length or width of the formed stopes, which is commonly observed in narrow deposit formations as utilized in this case study. Further, the application of the Mathews stability analysis to the optimization algorithm was deemed successful, as indicated by the good stope economic values and stable conditions at each stope wall.

8 Conclusions

One of the significant challenges in underground mine planning is determining the more layout, which involves the location, wall area, and stope size. The stope layout



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dictates the amount of material estructed, the meral content, and grade of minerals extracted, altimately determining the aggregate economic value of all extracted stopes. Addressing this challenge has been largely done through the development of optimization algorithms, mabling mining organizers to assess various stope layouts more efficiently. However, the involvement of numerous parameters renders optimization algorithms susceptible to suboptimal conditions, wherein the stope layout produced by the optimization process may not be the best solution achievable for a given case study. Among the multitude of parameters involved, tock conditions are one of the dominant parameters considered in determining the stope layout. The study's proposed algorithm tried to combine the steps of stope optimization with stability analysis using the Mathews Stability Chart. This was done so that a more complete design could be made, especially since more field data about the geomechanical properties was available

Integration is suggested by establishing dimensional constraint at the onset of optimization utilizing Mathews stability analysis. This involves incorporating geomechanical data, such as rock mass classification Q², factor A, factor B, and factor C. The outcomes demonstrate the effectiveness of this approach, evident in the optimization outcomes that yield superior economic value compared to employing fixed dimensions. Furthermore, the stability of the optimized stope walls affirms the effectiveness of stability analysis within the optimization algorithm, ensuring the project's financial feasibility. The results of optimization by this algorithm could serve as preliminary guidance for mine planners during the feasibility avaluation phase.

Author Contribution: Conceptualization, software, investigation, vioutization, writing --invice and odding, and project alone [5] even Dura Permi methodology, wildditon, ferond undy its and writingprocess and editing. Tit Karnik, validation, feroneux, writing --even and editing, supervision, and funding acquiritient. Bud Softwareson supervision, measures, redidence, and writing--review and editing Medamail Net Heriowski.

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Declarations

Computing Interests. The authors declare to computing interests

References

- Massikolog C, Trumman E, Waran WJ (2001) Executing the Mathews stability graph for open-stope design. Min Technol 110:27–38. https://doi.org/10.1079/nam.2008.110.127
- Lu A, Yin C, Zhang N (2019) Analytic stress solutions for a fixed initial transit under fractional slip cost act conditions. Tax J Mach A Solds 75: 03-20 mpc Adva org/10.1016/J.1.11000010215911, 101503.000
- Vital OPM, Calostino TH, Jieher A (2018) Shallow neural semidepent with provisite primipal stores drawtime: analytical solation and 3d Gene reflects. *Tunn Undergo Space* Technol 99 268– 203. https://doi.org/10.1016/J.TUST.2010.04.005
- Napo-García GF, Cânaso TR, and Navarov Torres VF (2015) Optimization of room-and-pillar damensions using an annual sumerital models. Int J Min Sci Technol 29. https://doi.org/10.1010/J. LMNT 2000 02 001.
- Abdullah WK, Abreut HM, Huttu MA (2019) Numerical modelling of singled stops obtained in a labelar scepty-dispate depoint. Generation General, 14:41–51. https://www.org/10.1069/11469025. 2018.0548056
- Mansah T (2023) A binary integer lanar programming model for optimizing a binary integer linear programming model for optimizing metergeneral ways layout underground scope layout trays. Autocharming and educations, these MTS.
- Lowson AR, Biptinweld ZT (2013) Critical assessment of HMR based runnel design practices: a practical ergineer's approach MME.
- Kang Z et al (2019) Optimisation calculation of stope structure parameters based on Mathews stabilization graph method. J Vibrang 21:1127–1230
- Wang D'et al (2021) Stope stability assessment by the Mathews-Port in method: a case-study of open stoping in soli rusk mass worker conditions of secondary stress field. JOP Profishing. https://dx.org/10.1010/J.755.1713-06421/021011
 List R, Zhoo Y, Zhang P, Lin F, Yang T (2021) Stope structure
- Lin B, Zhao Y, Zhang P, Lin F, Yang T (2021) Stope structure resolutions busied on the change model drives by microscionic data and Mathews stability diagram method in Xiadian gold mice, Genma Nucl. Has Risk 12, 1616–1637, https://doi.org/10. 1000/03677915.23221.3041.308
- Janiszowski M, Paniow S, Rimin M (2011) Industry survey an the current state of stope design methods in the underground mixing sector Energies (Essai) 15:140. https://doi.org/10.3086/ en.15010240
- Nikbin V, Atace-pour M, Shahriar K, Burnshimian Y, Mirflassant SA (2010) Stope brandup optimization: a multimatical model and efficient heimatics. Resour Policy 82:513–526. https://doi.org/10.1016/J.42550120201.2014.10007
- Basiri Z (2018) Stopes layout and productive scheduling sprimitinian insuffered stoping mining. Dissertation, University of Alberta.
- Nella G, Gurnube M, Massate D, Bur X (2016) Stope optimizer (ion with seriiral autorecity constraints. Optim Eng 17:813–852 https://doi.org/10.10077/11061-016-0521-6
- Wilson B (2029) Hearinky stachastic stope layest optimization University of Alberta, Thesis
- Parnado e Paria M, Dientradegoulos R, and Lopes Pinto CL (2022) Imaginatol stochastic optimization of steps design and loop spent anderground mine production scheduling. Research Policy 78: https://doi.org/10.1016/j.pescopped.1022.102918.
- Furnado e Faria MA, Dimitralopoulos R, Phres C (2022) Stoduo to stope de ign optimisation under grade meertanty and dynamic development costs. Int J Mar Bochan Environ 36:80– 103. https://doi.org/10.1030/17450930.2021.1068707

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- Sani YA, Kamzai M (2021) Suthinval stope layout planning through a groupy heavistic approach based on dynamic pregramming. J Oper Res Sev 72:234–303. https://doi.org/10.1000/ 01007662.2010.5700179
- Toloum K., Mossawi E. Tabrini ABB, Alexi P. Barcari AA (2021) An optimisation approach for anomaloy-based longterin production scheduling is open-ph mines using meta-benmics algorithms. Int J Min Rectam Environ 35:115–140. https:// doi.org/10.1060/17400406.2020.17773110
- Langhari A. Dimitrakopoulos B. (2020) Hyper-featuritie approaches for strategic raise planning suder assertatory. Comput Oper Res 113:304598. https://doi.org/10.1016/j.cm/2018.11. 010
- Oyunic J, Trang DS (1995) Economic optimization of stope geometry using separable programming with special bunch and bound indusingses. Mod B University 23. Riddle M (1977) Dynamic programming solution of a black caving rating layout. 367–180: https://www.opensine.org/documors/layout.com/parameters/para
- South 28, 1997 (1997) and programming contention of a function on the dynamic sprogramming evolutions of a block-cavity-interlayout Accessed. Jan. 20, 2023.
- Doranome J, De Foncport C, Fradous H (1986) Generational comlocity model computer optimization of peoples from different underground mining mathods. 583–580
 Christian MTNB, Delias EP, Mallet JL (1988) GEOCAD: an after-
- Cheirnenoff NM, Deline EP, Maller R. (1989) GEOCAD: an aftermation cail and artificial intelligence used that helps moving boos geological resources to minutelike reserves. Publ by 3 sc of Mining Engineers of AIME.
- 25. Allord C (1995) Optimization in underground mine design.
- Cave yet 1 (2011) Multiple gets floating stope process.
 Anale Pour M (2004) Opination in of stope limits using a learning to approach. Inc: Mining Metal Trans Syst. A: Min Technol 113: https://doi.org/10.1179/07179801239001290.
- Topal F, Sen J (2000) A new algorithm for single boundary optirization. J Gast Sci Erg 35:117–119. https://doi.org/10.1007/ s12004-010-0201-y
- Sandampake DSS, Topal E, Ali Aoad MW (2015) A heuristic approach sis-permitidesign of an mediaground using steps layous. Appl Soft Computer 30:599–603. https://doi.org/10.1010/LASOC, 2015.00.000
- Bai X, Mancing D, Sanon B (2013) Underground surpe optimination with network flow method. Comput Georgi 52:361–371, https://doi.org/10.1016/j.cagon.2012.10.010
- Samdanayake DSS, Topal E, Asad XWA (2015) Designing an optimal steps typon the underground mixing based on a humistic algorithm. Int J Min Sci Toubied 25:767–772. https://doi.org/10. 1016/j.ppnt.2015.07.011
- Sandamyske DS5 (2014) Stope brandary optimisation in underground mixing based on a bearing approach. Discention. Carfa University
- University
 Sari YA, Kasund M (2010) A planning approach for polymetallic mission using a subset of opting technique with pillin and offmate riope brains. Eng Optins 52:992–946. https://doi.org/10.1000/ 00062552.2010.16204759
- Sari YA, Kannad M (2021) Clustering-based iterative approach to stopp layout optimization for tablevel stoping. J South APPling Min. Metall 21:377–108, https://doi.org/10.1712/02411-97177 12072001

- 35. Kanaral M. San' YA (2020) Underground raine planning for dope-
- based methods. AIP Publishing LLC 18(10603):6000787
 Villatha Minamares ME, Korwel M (2017) Residue topo lopan optimization accomming for variable stops dimensions and attation management. Art Mis Miner Eng 81–18. https://doi.org/10.
- 1304/03/0402 2017 a 02000 27. How J. Xu C. Dowd PA, Li G (2019) Integrated optimisation of stopu branchery and access toyout for undarground rulning optimtions. Min Tuchnel 120:193–205. https://doi.org/10.1000/25728 146.2020.1017020
- Y. Nikhin E. Mardansh M. Waqar A. Asad. E. Topal 2022 Potern search method for accelerating more boundary optimization prellow in underground mining operations. https://doi.org/10.1080/ 1040921552.2021.1092800
- Damardi A, Hamidi JK, Mennevi A (2023) Determination of soblevel support large a network law algorithm and the MIMR chamiltening system. Resear Policy 20 (2022), https:// doi.org/10.1036/J.RESOURPOL.2022.105200
- Lastischer EH (1990) A geomechante eclosofication system for the initiag of each mass in mire design. J South Alt Initi Min Metall 00:257–273
- Potstn Y, Hadpigeorgies J (2001) The itability graph method. Underground Mining Methods 66:513–520
- Mashewe, K. E. Hoek, D. C. Wyllin, and S. H. V. Stewart (1980) Prediction of stable excavation symmetry muticing at depths below 1000 metrics in hard read. Inter Journal of a stable stability interfaces and stability (Interfaces) (Processing Stability (Interface 1053) (22 Accessed), Mar. 02, 2020
- Potytin Y (1988) Empirical open stope durign in Canada. Unloarsity of British Calumbia.
 Nadoren SD (1992) Cable support pedalisms for hand each mine
- Nadoon SD (1992) Cable support geidelines for hard each min operations. University of Bettels Columbia
- Nikisia A, Thohma T, Neingo P (2008) A tensive of andergoward steps boundary optimization algorithms. Revear Parkey 56:59:69. https://doi.org/10.1016/2.00550000001.2007.12.004
- Prastyce E (2010) Fractal model and classical black model in ore reserve estimation: a comparison. Black Geningi Dav Perturbanpar 20:115–120
- Parminto SPJ, Sacoda T, Watsimuna RK, Maroni K, Maroni K (2017) Inflament of stope design on stability of langing wall declate in Citating underground pdd mine. In J Genni 04(1-8): https://doi.org/10.4255/1jj.2013.1004001
- Budi X, Wattimena KK. Andramo A, Manud K (2009) Determisation of stage geometry in jointed took states at Proglem indergrowth gold mine. Int JJCRM 563–66. https://doi.org/10.11187/ 100905-663

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